

# Thermal and Quantum critical Properties of overdoped $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

T. Schneider<sup>1,\*</sup>

<sup>1</sup>*Physik-Institut der Universität Zürich, Winterthurerstrasse 190, CH-8057, Switzerland*

We analyze the extended superfluid density data of Božović *et al.* taken on homogenous thin films to explore the critical properties of the thermal (TSM) and quantum superconductor to metal (QSM) transitions in overdoped  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ . The temperature dependence reveals remarkable agreement with d-wave BCS behavior, where  $\sigma \propto 1/\lambda^2$ . No sign of the expected KTB transition is observable. We show that the critical amplitude  $\rho_{s0} = \rho_s(T)/(1 - T/T_c)$  scales as  $T_c \propto \rho_{s0}^{1/2}$  and with that as  $T_c \propto \rho_s(0)$ , empirically verified by Božović *et al.* Together with additional evidence for BCS behavior in overdoped  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  we show that the scaling relation  $T_c \propto \lambda(0)^{-1} \propto \Delta(0) \propto \xi(0)^{-1} \propto H_{c2}(0)^{-1/2} \propto \lambda_0^{-1} \propto \Delta_0 \propto \xi_0^{-1} \propto H_{c20}^{-1/2}$  applies, by approaching the QSM and TSM transitions. This differs from the dirty limit behavior  $\lambda(0) \propto \xi(0)^{1/2}$ , maintains the  $T_c$  independence of the Ginzburg-Landau parameter  $\kappa = \lambda_0/\xi_0$ , and puts a stringent constraint on the effect at work. We notice that potential candidates appear to be the nonlocal effects on the penetration depth of clean d-wave superconductors explored by Kosztin and Leggett.

PACS numbers:

Considering thin films and adopting the phase transition point of view the thermal (TSP) and quantum (QSM) superconductor to metal transitions are expected to fall onto the respective two dimensional (2D) xy- universality class. Accordingly the TSP transition should exhibit the characteristic Kosterlitz-Thouless-Berezinski (KTB) behavior<sup>1</sup>, including a discontinuous drop in the superfluid stiffness  $\rho_s(T) \propto d/\lambda^2(T)$  from<sup>2</sup>,

$$\frac{d}{\lambda^2(T_c^-)} = \frac{32\pi^2}{\Phi_0^2} k_B T_c \simeq 1.017 T_c, \quad (1)$$

to zero, with  $d/\lambda^2(T_c^-)$  in  $\text{cm}^{-1}$  and  $T_c$  in K.  $\Phi_0 = hc/2e \simeq 2.07 \times 10^{-7} \text{erg}^{1/2} \text{cm}^{1/2}$ .  $d$  denotes the film thickness. In a homogenous film this relationship applies if  $d/\lambda^2(T)$  is measured at zero frequency and  $\lambda^2(T)/d$  is large compared to the lateral extent of the film. Otherwise there is a rounded BKT-transition. In particular, if the lateral extent of the homogeneous regions  $L$  is finite the correlation length cannot grow beyond  $L$ . Similarly, there is no phase transition at finite frequency because the frequency scales as  $1/\omega \propto \xi^{z_{cl}} \propto L^{z_{cl}}$  and so gives rise to a smeared Nelson-Kosterlitz jump<sup>3</sup>. Noting that  $d/\lambda^2(T)$  and  $\rho_s(T)$  are related by

$$\frac{d}{\lambda^2(T)} = \frac{4\pi\alpha k_B}{\hbar c} \rho_s(T), \quad (2)$$

where  $k_B$  is Boltzmann's,  $\hbar$  Planck's,  $\alpha$  the fine structure constant, and  $c$  speed of light. From Eqs.(1) and (2) we obtain for the BKT line the relation

$$T_c = \frac{\Phi_0^2 \alpha}{8\pi \hbar c} \rho_s(T) \simeq 0.39 \rho_s(T), \quad (3)$$

with  $T_c$  and  $\rho_s$  in K. Fig. 1a shows data taken from Božović *et al.*<sup>4</sup> of the superfluid stiffness  $\rho_s(T)$  for a selection of overdoped thin  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ . S denotes the selected samples. The dashed line is the KTB transition temperature where at the intersection with  $\rho_s(T)$

the universal jump should occur. Obviously there are no signs for the universal jump at the respective KTB transition temperatures. In fact, the rather sharp TSM transitions occur considerably above the respective KTB transition temperatures and reveal remarkable consistency with the mean-field temperature dependence

$$\begin{aligned} \rho_s(T) &= \rho_{s0}(T_c) t = \frac{4\pi\alpha k_B}{\hbar c} \frac{d}{\lambda^2(T)} \\ &= \frac{4\pi\alpha k_B}{\hbar c} \frac{d}{\lambda_0^2(T_c)} t = \frac{4\pi\alpha k_B}{\hbar c} \frac{d}{\lambda_0^2(T_c)} t, \end{aligned} \quad (4)$$

where  $t = (1 - T/T_c)$ . Note that such an extended linear  $t$  dependence is a characteristic of feature of d-wave BCS superconductors where thermal phase fluctuations, driving the BKT transition, are fully neglected<sup>5</sup>. The solid lines in Fig. 1a are fits to Eq. (4). The resulting fit parameters,  $\rho_{s0}$  and  $T_c$ , are collected in Fig. (2) and yield the scaling plot shown in Fig. 1b. Obviously, when the QSM transition is reached, the data fall onto a straight line and thus confirm the BCS d-wave scenario very impressively. Given the evidence for the irrelevance of phase fluctuations, another unexpected empirical fact emerges from Fig. 2, depicting  $T_c(\rho_{s0}^{1/2})$ . The dashed line  $T_c = f\rho_{s0}^{1/2}$ , describing the approach to the QSM transition rather well, agrees with the zero temperature counterpart  $T_c \propto \rho(0)^{1/2}$ , verified by Božović *et al.*<sup>4</sup>. In fact, this relationship contradicts the general belief that the measured penetration depth corresponds to the London penetration depth, whereupon

$$\rho_s(0) \propto 1/\lambda(0)^2 \propto n, \quad (5)$$

applies, where  $n$  is fully determined by the shape of the Fermi surface<sup>12</sup>. For circular or spherical Fermi surfaces it corresponds to the electron density in the normal state. As shown by angle-resolved photoemission experiments, this is not the case in overdoped  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ <sup>11</sup>. However, the result is determined by the shape of the Fermi

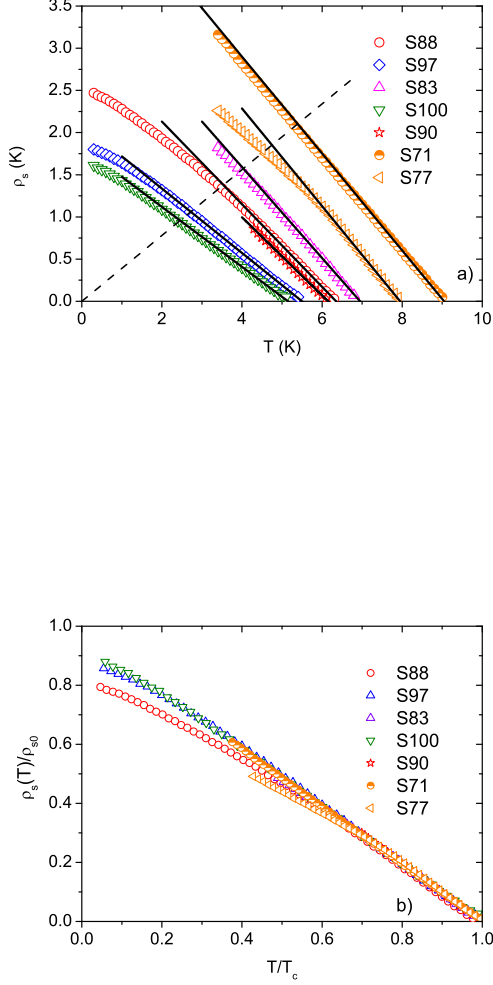


FIG. 1: a) Superfluid stiffness  $\rho_s(T)$  for a selection of overdoped thin films taken from Božović *et al.*<sup>4</sup>. S denotes the selected samples. The dashed line marks the KTB transition temperature where the universal jump in the superfluid stiffness should occur (Eq. (3)). The solid lines are fits to Eq. (4) yielding for  $\rho_{s0}$  and  $T_c$  the estimates collected in Fig. (2). b) Scaling plot  $\rho_s(T)/\rho_{s0}$  vs.  $T/T_c$ .

surface and independent of  $T_c$ . Therefore even the more general treatment of the zero temperature penetration depth is incompatible with the empirical relation

$$\begin{aligned} T_c &\propto \rho_s(0)^{1/2} \propto \rho_{s0}^{1/2} \\ &\propto 1/\lambda(0) \propto 1/\lambda_0, \end{aligned} \quad (6)$$

emerging from Fig. 2.  $T_c \propto \rho_s(0)^{1/2}$  was verified by Božović *et al.*<sup>4</sup> and  $T_c \propto \rho_{s0}^{1/2}$  follows from our analysis

shown in Fig. 1, yielding  $\rho_{s0}(T_c)$  depicted in Fig. 2. Note that the observation of a diminishing  $\rho_s(0)$  in the overdoped regime is confirmed by earlier measurements.<sup>6–10</sup> In addition there is the hyperscaling prediction<sup>3,13</sup>

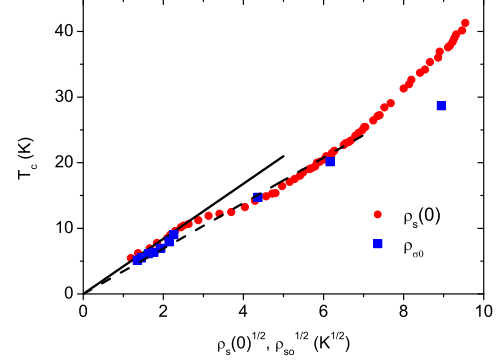


FIG. 2: Estimates for the critical amplitude  $\rho_{s0}(T_c)$ , shown as  $T_c$  versus  $\rho_{s0}^{1/2} \propto 1/\lambda_0^{1/2}$ , derived from the fits shown in Fig. 1a. The dashed line is  $T_c = f\rho_{s0}^{1/2}$  with  $f = 3.46 \text{ K}^{1/2}$ . For comparison we included  $T_c$  versus  $\rho_s(0)^{1/2}$  of Božović *et al.*<sup>4</sup>  $\rho_s(0) \propto 1/\lambda(0)^2$  is the zero temperature superfluid stiffness and the solid line is  $T_c = g\rho_s(0)^{1/2}$  with  $g = 4.2 \text{ K}^{1/2}$ .

$$T_c \propto \rho_s(0)^{z/(D+z-2)}, \quad (7)$$

where  $z$  denotes the dynamic critical exponent of the quantum transition. It is applicable whenever a phase transition line  $T_c(x)$  exhibits a critical endpoint at  $x = x_c$  of the tuning parameter  $x$ . Here the transition temperature vanishes and a quantum phase transition occurs. In thin films ( $D = 2$ ) it yields  $T_c \propto \rho_s(0)$ , irrespective of the value of the dynamic critical exponent  $z$ , and with that it contradicts the empirical relation (5). Given the unexpected empirical relation (5), the evidence for d-wave BCS behavior in the temperature dependence of  $\rho_s(T)$ , and the resulting irrelevance of phase fluctuations, it is suggestive to explore the BCS scenario further. Considering the observable  $O(T)$  with critical amplitude  $O_0$  and  $O(T = 0) = O(0)$ , including the gap  $\Delta$ , the correlation length  $\xi$  and the upper critical field  $H_{c2}$ , the critical amplitudes should scale as<sup>3,14</sup>

$$\begin{aligned} T_c &\propto \Delta_0 \propto \xi_0^{-1} \propto H_{c20}^{-1/2} \\ &\propto \Delta(0) \propto \xi(0)^{-1} \propto H_{c2}(0)^{-1/2}, \end{aligned} \quad (8)$$

Here,  $\lambda(T) = \lambda_0 t^{-1/2}$ ,  $\Delta(T) = \Delta_0 t^{1/2}$ ,  $\xi(T) = \xi_0 t^{-1/2}$ ,  $H_{c2} = H_{c20} t \propto 1/\xi^2$ , where  $t = 1 - T/T_c$ . These scaling

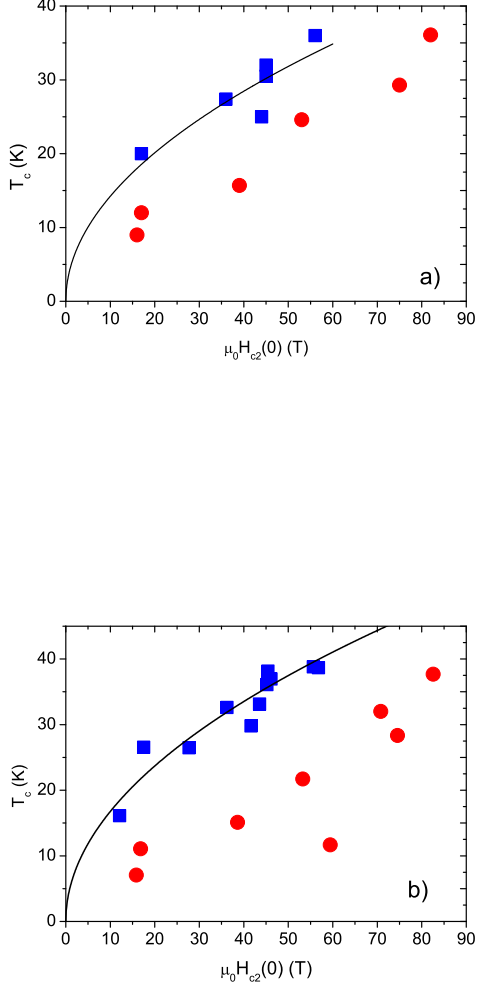


FIG. 3:  $T_c$  versus  $H_{c2}(0)$ . a) Taken from Y. Wang and H.-H. Wen as derived from specific heat measurements on  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  single crystals<sup>15</sup>. (■: overdoped regime; ●: underdoped regime). The line is  $T_c = aH_{c2}(0)^{1/2}$  with  $a = 4.5\text{KT}^{-1/2}$ . b) Taken from Rourke *et al.* as derived from magnetoresistance measurements  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  single crystals<sup>17</sup>. (■: overdoped regime; ●: underdoped regime). The line is  $T_c = aH_{c2}(0)^{1/2}$  with  $a = 5.3\text{KT}^{-1/2}$ .

relations follow from  $\Delta(T) \propto 1/\xi(T) \propto H_{c2}(T)^{-1/2}$ , and  $\Delta(0) \propto \Delta_0 \propto T_c$ .

In order to clarify their consistency with experimental facts, we need the  $T_c$  dependence of  $H_{c2}$  and the gap  $\Delta$ . In Fig.3a we depicted  $T_c$  versus  $H_{c2}(0)$  taken from Y. Wang and H.-H. Wen derived from specific heat measurements on single crystals<sup>15</sup>. Although the data are

rather sparse the flow to the QSM transition is revealed, and consistency with the BCS scaling law (8) can be anticipated. Consistency also emerges from Fig.3b, depicting the magnetoresistance measurements of Rourke *et al*<sup>17</sup> on single crystals. Quantitative agreement with BCS theory for a d-wave superconductivity in overdoped cuprates stems from heat transport measurements in  $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\vartheta}$ <sup>18</sup>.

Fig. 4 shows the doping dependence of the gap in terms of  $\Delta(0)/k_B T_c$  versus  $x$  for  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  taken from Wang *et al*<sup>16</sup>, derived from low temperature specific heat measurements on single crystals. The dotted line marks the d-wave BCS value  $\Delta(0)/k_B T_c \simeq 2.14$ . Note that for  $x \gtrsim 0.19$  the data are consistent with the scaling form (8) and in addition with d-wave BCS (weak coupling) superconductivity. More compelling evidence for this scenario stems from heat transport measurements in  $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\vartheta}$ <sup>18</sup>. Potential candidates appear to be

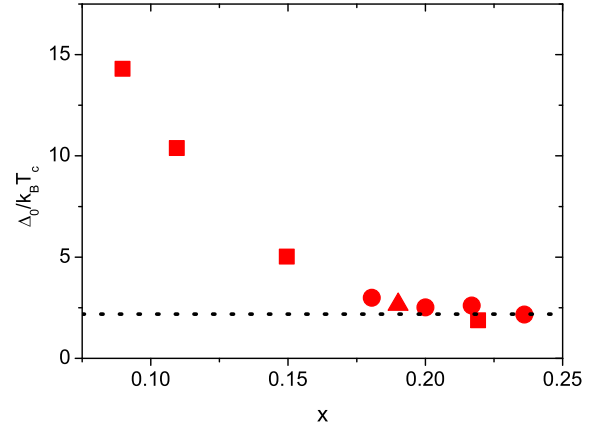


FIG. 4: Gap  $\Delta(0)/k_B T_c$  versus  $x$  for  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  taken from Wang *et al*<sup>16</sup>. The dotted line marks the limiting d-wave BCS value  $\Delta(0)/k_B T_c = 2.14$ . Note that for  $x \gtrsim 0.19$  the data are consistent with the scaling form (8).

the nonlocal effects on the penetration depth of clean d-wave superconductors explored by Kosztin and Legget<sup>19</sup>. Taking the nonlocal effects into account they obtained for a specular boundary the relation

$$\frac{\lambda_{spec}(0)}{\lambda_L(0)} = 1 + \frac{\pi\sqrt{2}}{16} \frac{\xi(0)}{\lambda_L(0)}, \quad (9)$$

where  $\lambda_L(0)$  is the zero temperature London penetration depth. Considering the limit  $\xi(0)/\lambda_L(0) \gg 1$  they expected no significant corrections at zero temperature. This differs from the present case where the QSM transition is approached,  $\xi(0) \propto 1/T_c$  tends to diverge and the limit  $\xi(0)/\lambda_L(0) \gg 1$  is attained.

In this case the nonlocal corrections dominate and Eq. (9) reduces, in accordance with the empirical relation ((6), to  $\lambda_{spec}(0) \propto \xi(0) \propto 1/T_c$ . To estimate the  $T_c$  regime where this limit is reached we calculate the  $T_c$  dependence of  $\xi(0)$ . With  $\xi(0)^2 = \Phi_0 / (2\pi H_{c2}(0))$  and  $T_c = a H_{c2}(0)^{1/2}$  with  $a = 5 \text{ K T}^{-1/2}$  (see Fig.3) we obtain  $\xi(0) \simeq 900/T_c \text{ \AA}$  with  $T_c$  in K. Consequently  $\xi(0)/\lambda_L(0) \gg 1$  is fulfilled if  $\lambda_L(0) < 900/T_c \text{ \AA}$ , where  $\lambda_L(0)$  is fully determined by the properties of the Fermi surface.

In summary, we analyzed the temperature dependence of the superfluid stiffness of selected overdoped  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  thin films, using the data of Božović *et al.*<sup>4</sup>. The temperature dependence did not exhibit any sign of the expected KTB transition. We observed remarkable consistency with a linear temperature dependence, pointing to d-wave BCS behavior. On the other hand, we have shown that the critical amplitude  $\rho_{s0}$  and the zero temperature counterpart  $\rho_s(0)$  adopt essentially the same  $T_c$  power law dependence. This contradicts the standard result, whereupon  $\rho_s(0)$  remains finite and is de-

termined by Fermi surface properties. Exceptions include dirty limit superconductors, where  $\rho_s(0) \propto T_c$ . Moreover we have shown that the  $T_c$  of  $\Delta(0)$  and  $H_{c2}(0)$  are consistent with the expected BCS behavior, as it should be, because both are proportional to some power of the correlation length  $\xi(0)$ . Noting that the correlation length is the essential length scale the measurements of Božović *et al.*<sup>4</sup> imply that  $\rho_s(0) \propto \rho_{s0} \propto \xi(0)^{-2} \propto \xi_0^{-2}$  holds by approaching the TSM and QSM transitions down to 5 K. We observed that potential candidates appear to be the nonlocal effects on the penetration depth of clean d-wave superconductors explored by Kosztin and Leggett<sup>19</sup>. It should be kept in mind, that closer the QSM transition ( $T_c = T = 0$ ) quantum fluctuations are expected to modify the outlined scaling behavior and uncover the difference between bulk and thin film samples.

Acknowledgements Karl Alex Müller I am grateful for our friendship dating back to discussions on high temperature superconductivity in metallic hydrogen in 1970.

- 
- \* Electronic address: tschnei@physik.uzh.ch
- <sup>1</sup> J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973); V. L. Berezinskii, Zh. Eksp. Teor. Fiz. 61, 1144 (1971) [Sov. Phys. JETP **34**, 610 (1972)].
  - <sup>2</sup> David R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. **39**, 1201 (1977).
  - <sup>3</sup> T. Schneider and J. M. Singer, *Phase Transition Approach to High Temperature Superconductivity* (Imperial College Press, London, 2000).
  - <sup>4</sup> I. Božović, X. He, J. Wu., and A. T. Bollinger, Nature **536**, 309 (2016).
  - <sup>5</sup> R. Prozorov, Supercond. Sci. Technol. **21**, 082003, (2008).
  - <sup>6</sup> C. Niedermayer *et al.*, Phys. Rev. Lett. **71**, 1764 (1993).
  - <sup>7</sup> C. Bernhard *et al.*, Phys. Rev. B **52**, 10488 (1995).
  - <sup>8</sup> J. P. Locquet *et al.*, Phys. Rev. B **54**, 7481 (1996).
  - <sup>9</sup> C. Panagopoulos *et al.*, Phys. Rev. B **67**, 220502 (2003).
  - <sup>10</sup> T. R. Lemberger *et al.*, Phys. Rev. B **83**, 140507 (2011).
  - <sup>11</sup> R. Razzoli *et al.*, New Journal of Physics **12**, 125003 (2010)
  - <sup>12</sup> B. S. Chandrasekhar and D. Einzel, Ann. Phys. (Leipzig), **2**, 535 (1993).
  - <sup>13</sup> K. Kim and P. B. Weichman, Phys. Rev. B **43**, 13583 (1991).
  - <sup>14</sup> A. L. Fetter and J. D. Walecha, *QUANTUM THEORY OF MANY PARTICLE SYSTEMS* (Mac Graw Hill Book Company, New York, 1971)
  - <sup>15</sup> Y. Wang and H.-H. Wen, EPL **81**, 5700, (2008).
  - <sup>16</sup> Yue Wang, Jing Yan, Lei Shan, and Hai-Hu Wen, Phys. Rev. B **76**, 064512 (2007).
  - <sup>17</sup> Patrick M. C. Rourke *et al.*, NATURE PHYSICS, **7**, 455 (2011).
  - <sup>18</sup> C. Proust *et al.*, Phys. Rev. Lett. **89**, 147003 (2002).
  - <sup>19</sup> I. Kosztin and A. J. Leggett, Phys. Rev. Lett. **79**, 135 (1997).